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SYSTEM OF ARBITRARILY ORIENTED LONGITUDINAL SHEAR CRACKS IN AN ELASTIC SOLID

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The following problems on determining the stresses around rectilinear longitudinal shear cracks are examined by the method of singular integral equations: a system of arbitrarily arranged cracks in an unbounded or semi-bounded solid, a periodic system of cracks of arbitrary orientation in infinite and semi-infinite spaces.

The simply-connected domain is usually considered in the investigations [1—9] devoted to a study of the stress distribution around longitudinal shear cracks, when the solution of the problem can be obtained by conformal mapping. If the domain occupied by the solid is multiconnected, then the existing solutions are limited to comparatively simple cases of collinear [1 – 3] or parallel [2—5, 8, 9] cracks.

The problem of determining the stresses in an infinite solid containing arbitrarily arranged rectilinear longitudinal shear cracks is reduced below to a system of integral equations in the general case. This permits the solution of a number of new problems of mathematical theory of cracks. The appropriate problems of plane elasticity theory were studied by the same method in [10, 11].

Let us note that the integral equations of problems for longitudinal shear cracks agree with the integral equations of the appropriate plane problems of heat conduction for a body with heat-insulated cracks.

1. It is known [1, 6] that the solutions of longitudinal shear problems reduce to the determination of an analytic function $F(z)$ of the complex variable $z = x + iy$ in the domain occupied by the solid. The stress components τ_{xz}, τ_{yz} and displacement w are hence determined in terms of $F(z)$ by means of the formulas (μ is the shear modulus)

$$\tau_{xz} - i\tau_{yz} = F(z), \quad w(x, y) = \frac{1}{\mu}\operatorname{Re} f(z), \quad F(z) = f'(z) \tag{1.1}$$

If a new coordinate system (x_1, y_1) is connected with the old (x, y) system by the relationship

$$z = z_1 e^{i\alpha} + z^\circ, \quad z_1 = x_1 + iy_1, \quad z^\circ = x^\circ + iy^\circ$$

and the function $F_1(z_1)$ plays the same part in the (x_1, y_1) system as does the function $F(z)$ in the (x, y) system, then we have

$$F_1(z_1) = e^{i\alpha} F(z_1 e^{i\alpha} + z^\circ) \tag{1.2}$$

Here x°, y° are coordinates of the origin in the (x_1, y_1) system in the old (x, y) system. Let there be a slit (crack) in a solid which is in the state of longitudinal shear along the strip $|x| \leqslant a$, $y = 0$. We examine the case when there are no stresses at infinity and the self-equilibrated load.

$$\tau_{yz}^+ = \tau_{yz}^- = p(x), \quad |x| \leqslant a \tag{1.3}$$

acts on the surfaces of the slit.

Let $2g(x) / \mu$ denote a discontinuity in the displacements upon going through the plane of the slit

$$2\mu^{-1} g(x) = w^+(x, 0) - w^-(x, 0), \quad |x| \leqslant a \tag{1.4}$$

Expressing the conditions (1.3) and (1.4) in terms of boundary values of the function $F_1(z)$ on the segment $|x| \leqslant a$, $y = 0$, we arrive at the conjugate problem

$$F_1^+(x) - F_1^-(x) = 2g'(x), \quad |x| \leqslant a$$

Hence, the piecewise-holomorphic function $F_1(z)$ which decreases at infinity is determined by a Cauchy-type integral [12]

$$F_1(z) = \frac{1}{\pi i} \int_{-a}^{a} \frac{g'(t)\, dt}{t - z} \tag{1.5}$$

Defining the stress τ_{yz} in the plane of the slit by means of (1.1) and equating it to the given load (1.3), we obtain a singular integral equation in the unknown function $g'(x)$.

$$\int_{-a}^{a} \frac{g'(t)\, dt}{t - x} = \pi p(x), \quad |x| \leqslant a$$

Taking into account that $g(-a) = g(a) = 0$, we find [13]

$$g'(x) = -\frac{1}{\pi \sqrt{a^2 - x^2}} \int_{-a}^{a} \frac{\sqrt{a^2 - t^2}\, p(t)\, dt}{t - x}$$

Hence, we determine the coefficient of stress intensity k_3 for a longitudinal shear crack (the upper signs refer to the right vertex of the crack, and the lower sign to the left vertex)

$$k_3^{\pm} = \mp \lim_{x \to \pm a} \left[\frac{\sqrt{a^2 - x^2}}{\sqrt{a}} g'(x) \right] = -\frac{1}{\pi \sqrt{a}} \int_{-a}^{a} \sqrt{\frac{a \pm t}{a \mp t}}\, p(t)\, dt$$

This formula has been obtained by other means in [6].

2. Let there be N rectilinear slits of width $2a_k$ $(k = 1, 2, \ldots, N)$ in an infinite solid referred to an (x, y, z) coordinate system whose antiplane deformation axis is directed along the z-axis. The centers of the slits O_k are determined by the coordinates $z_k^\circ = x_k^\circ + iy_k^\circ = d_k e^{i\beta_k}$. The origins of local (x_k, y_k) coordinate system are placed at the points O_k. The x_k-axes lie in the planes of the slits and make angles α_k with the x-axis (Fig. 1). There are no stresses at infinity, and the surfaces of the slits are loaded by the self-equilibrated forces

$$\tau_{y_k z}^{+} = \tau_{y_k z}^{-} = p_k(x_k), \quad |x_k| \leqslant a_k, \quad k = 1, 2, \ldots, N \tag{2.1}$$

The state of stress of a solid, caused by discontinuities $g_k(x_k)$ in the displacements on N strips $|x_k| \leqslant a_k$, $y_k = 0$ $(k = 1, 2, \ldots, N)$, is characterized bv the function

$$F_2(z) = \frac{1}{\pi i} \sum_{k=1}^{N} e^{-i\alpha_k} \int_{-a_k}^{a_k} \frac{g_k'(t)\, dt}{t - z_k}, \quad z_k = e^{-i\alpha_k}(z - z_k^\circ) \tag{2.2}$$

which is obtained by superposition of the stress functions (1.5) for single cracks by taking account of the conversion formula (1.2) for the passage to a new coordinate system.

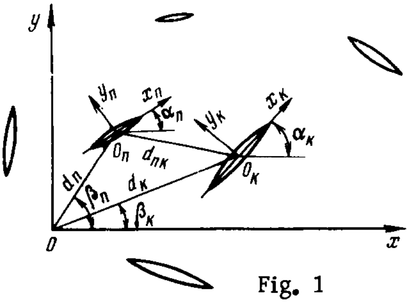

Fig. 1

The function $F_2(z)$ satisfies all the requirements of the problem except the boundary conditions (2.1) on the surfaces of the slits. By determining the stresses $\tau_{y_k z}$ in the planes of the slits and equating them to the given load $p_k(x_k)$, we obtain a system of N singular integral equations for the problem

$$\int_{-a_n}^{a_n} \frac{g_n'(t)\, dt}{t - x} + \sum_{k=1}^{N}{}' \int_{-a_k}^{a_k} K_{nk}(t, x)\, g_k'(t)\, dt = \pi p_n(x), \quad |x| \leqslant a_n, \tag{2.3}$$

$$n = 1, 2, \ldots, N$$

The symbol Σ' means that the term with the number of the row is eliminated during the summation. The kernels $K_{nk}(t, x)$ are defined by the relationships

$$K_{nk}(t, x) = \operatorname{Re}\left(\frac{e^{i\alpha_n}}{T_k - X_n} \right), \quad T_k = te^{i\alpha_k} + z_k^\circ, X_n = xe^{i\alpha_n} + z_n^\circ$$

Let us note that in the case of collinear cracks $(y_n^\circ = 0,\ \alpha_n = 0,\ n = 1, 2, \ldots, N)$ the system (2.3) agrees with the corresponding system of integral equations for cracks of a normal tension and transverse shear [10]. Hence it follows that the solution of the problem for any system of collinear transverse or longitudinal shear cracks can be obtained from the solution for cracks of normal tension by a simple change of symbols.

In the case of two parallel slits $(\alpha_1 = \alpha_2 = 0)$ of identical width $(a_1 = a_2 = a)$, loaded in such a way that $p_1(x) = p_2(-x) = p(x)$, the system (2.3) is converted into one integral equation

$$\int_{-a}^{a} \left[\frac{1}{t-x} + \frac{t+x+d\cos\beta}{(t+x+d\cos\beta)^2 + d^2\sin^2\beta}\right] g'(t)\,dt = \pi p(x), \quad |x| \leqslant a$$

Here $g'(x) = g_1'(x) = -g_2'(-x)$, d is the spacing between centers of the slits, and β is the angle between the plane of the slit and a line passing through the middle of the slits.

For a constant load $p(x) = \tau$, known integral equations can be obtained from the last equation in the case of parallel slits "not shifted" $(\beta = \pi / 2)$ [8] or "shifted" by the distance $2a$ $(d\cos\beta = 2a)$ [5].

We find the solution of the problem for large spacings between the slits. In this case, the kernels $K_{nk}(t, x)$ have the expansions (C_p^ν are binomial coefficients)

$$K_{nk}(t, x) = \sum_{p=0}^{\infty} \sum_{\nu=0}^{p} a_{nkp\nu} t^\nu x^{p-\nu} d_{nk}^{-p-1}, \quad d_{nk} e^{i\beta_{nk}} = z_n^\circ - z_k^\circ$$

$$a_{nkp\nu} = (-1)^{p+\nu+1} C_p^\nu \cos[(p-\nu+1)\alpha_n + \nu\alpha_k - (p+1)\beta_{nk}]$$

Following [10], we obtain the solution of the system of integral equations (2.3) in the form of the series

$$g_n'(x) = \sum_{p=0}^{\infty} g_{np}'(x)\lambda^p, \quad \lambda = \frac{2a}{d}, \quad a = \max\{a_n\}, \quad d = \min\{d_{nk}\}$$

$$g_{n0}'(x) = -\frac{1}{\pi\sqrt{a_n^2 - x^2}} \int_{-a_n}^{a_n} \frac{\sqrt{a_n^2 - t^2}\, p_n(t)\,dt}{t-x}, \quad g_{n1}'(x) = 0$$

$$g_{np}'(x) = \frac{1}{\pi\sqrt{a_n^2 - x^2}} \sum_{k=1}^{N}{}' \sum_{s=1}^{p-1} \sum_{\nu=1}^{s} H_{s-\nu}\left(\frac{x}{a_n}\right) \left(\frac{\varepsilon_{kn}}{2}\right)^{s+1} a_n^{-\nu} a_{nks\nu} \times$$

$$\int_{-a_k}^{a_k} t^\nu g_{k,p-s-1}'(t)\,dt, \quad p = 2, 3, \ldots$$

$$H_p\left(\frac{x}{a_n}\right) = \frac{1}{\pi a_n^{p+1}} \int_{-a_n}^{a_n} \frac{\xi^p \sqrt{a_n^2 - \xi^2}\,d\xi}{\xi - x}, \quad \varepsilon_{nk} = \frac{a_k d}{a d_{nk}} \leqslant 1$$

Knowing the functions $g_n'(x)$ by means of (2.2) and (1.1), we can determine the state of stress in the whole domain. Let us write the values of the stress intensity coefficients at the vertices of any of the cracks

$$k_{3n}^{\pm} = -\frac{1}{\pi\sqrt{a_n}} \int_{-a_n}^{a_n} \sqrt{\frac{a_n \pm t}{a_n \mp t}}\, p_n(t)\,dt + \frac{\lambda^2\sqrt{a_n}}{4} \sum_{k=1}^{N} \varepsilon_{nk}^2 a_{nk11} G_{k0} + \qquad (2.4)$$

$$\frac{\lambda^3 \sqrt{a_n}}{8} \sum_{k=1}^{N}{}' \varepsilon_{nk}^2 \left(\varepsilon_{nk} a_{nk22} G_{21} \pm \frac{1}{2} \varepsilon_{kn} a_{nk21} G_{k0}\right) +$$

$$\frac{\lambda^4 \sqrt{a_n}}{32} \sum_{k=1}^{N}{}' \varepsilon_{nk}^2 \left[- a_{nk11} \sum_{r=1}^{N}{}' \varepsilon_{kr}^2 a_{kr11} G_{r0} + \varepsilon_{kn}^2 a_{nk31} G_{k0} \pm\right.$$

$$\left.\varepsilon_{nk} \varepsilon_{kn} a_{nk32} G_{k1} + \varepsilon_{nk}^2 a_{nk33} (G_{k0} + 2G_{k2})\right] + O(\lambda^5)$$

Here

$$G_{ks} = \frac{1}{\pi a_k^{s+2}} \int_{-a_k}^{a_k} t^s \sqrt{a_k^2 - t^2}\, p_k(t)\, dt \tag{2.5}$$

The formulas (2.4) yield the solution of the problem for any N for the arbitrary load (1.3). In particular, in the case of two equal cracks ($N = 2$, $a_1 = a_2 = a$) whose surfaces are load-free, for a given homogeneous shear $\tau_{yz}^{\infty} = \tau$ at infinity, we have

$$k_{3n}^{\pm} = \tau \sqrt{a} \left\{\cos \alpha_n + \frac{\lambda^2}{8} \cos \alpha_k \cos (\alpha_n + \alpha_k - 2\beta) \mp \right.$$

$$(-1)^k \frac{\lambda^3}{16} \cos \alpha_k \cos (2\alpha_n + \alpha_k - 3\beta) +$$

$$\frac{\lambda^4}{64} \left\{\cos \alpha_n \cos^2 (\alpha_n + \alpha_k - 2\beta) + \right.$$

$$\left.\left.\frac{3}{2} [2 \cos (3\alpha_n + \alpha_k - 4\beta) + \cos (\alpha_n + 3\alpha_k - 4\beta)] \cos \alpha_k\right\}\right\} + O(\lambda^5)$$

$$\beta = \beta_{21} = \beta_{12} + \pi \quad (n = 1, k = 2 \quad \text{or} \quad n = 2, k = 1)$$

3. We consider the centers of the slits to be on the x-axis, the spacing between the centers of adjacent slits to be the constant d ($z_k^{\circ} = kd$, $k = 0, \pm 1, \pm 2, \ldots$), the lengths and angles of slope of the slits to be identical ($a_k = a$, $\alpha_k = \alpha$). Under the assumption that the same load ($p_k(x) = p(x)$) is applied to all the slits and the number of slits tends to infinity, we obtain a periodic system of longitudinal shear cracks in an infinite solid. Hence $g_k'(x) = g'(x)$. After summation we find from (2.2)

$$F_3(z) = \frac{1}{id} \int_{-a}^{a} \operatorname{ctg} \frac{\pi}{d} (te^{i\alpha} - z)\, g'(t)\, dt$$

By satisfying the boundary condition on the surface of any of the slits we arrive at a singular integral equation in the unknown function $g'(x)$

$$\int_{-a}^{a} K(t - x)\, g'(t)\, dt = \pi p(x), \quad |x| \leqslant a \tag{3.1}$$

$$K(x) = \frac{\pi}{d} \operatorname{Re} \left(e^{i\alpha} \operatorname{ctg} \frac{\pi x e^{i\alpha}}{d}\right)$$

from which we find integral equations for a periodic system of collinear ($\alpha = 0$) or parallel ($\alpha = \pi / 2$) slits

$$\frac{1}{d} \int_{-a}^{a} g'(t) \operatorname{ctg} \frac{\pi (t - x)}{d}\, dt = p(x), \quad |x| \leqslant a \quad (\alpha = 0) \tag{3.2}$$

$$\frac{1}{d}\int_{-a}^{a} g'(t)\,\mathrm{cth}\,\frac{\pi(t-x)}{d}\,dt = p(x), \quad |x| \leqslant a \quad \left(\alpha = \frac{\pi}{2}\right) \tag{3.3}$$

This last equation has been obtained earlier, in [3], by other means. The solutions of (3.2) and (3.3) are easily found in closed form [14].

In particular, we have for the stress intensity coefficient in the case of a periodic system of parallel cracks

$$k_3^{\pm} = -\int_{-a}^{a} R^{\pm}(t)\,pt\,dt \tag{3.4}$$

$$R^{\pm}(t) = \left(\frac{\pi d}{2}\,\mathrm{sh}\frac{2\pi a}{d}\right)^{-1/2}\left(\mathrm{th}\,\frac{\pi a}{d} \pm \mathrm{th}\,\frac{\pi t}{d}\right)^{1/2}\left(\mathrm{th}\,\frac{\pi a}{d} \mp \mathrm{th}\,\frac{\pi t}{d}\right)^{-1/2}$$

If concentrated forces Q are applied at a point $x = x_0$ on opposite surfaces of a crack, i.e. $p(x) = -Q\delta(x - x_0)$ ($\delta(x)$ is the delta function), then we obtain from (3.4)

$$k_3^{\pm} = QR^{\pm}(x_0) \tag{3.5}$$

For a constant load on the crack $p(x) = -\tau$, we arrive at the known result [1–3]

$$k_3 = \tau\sqrt{\frac{d}{\pi}\,\mathrm{th}\,\frac{\pi a}{d}} \tag{3.6}$$

The stress intensity coefficient k_3 for a periodic system of collinear cracks under analogous loads can be determined by means of (3.4) – (3.6) if the hyperbolic functions in these latter are replaced by the corresponding trigonometric functions.

Making the change of variable $t = \xi - a$, $p(\xi - a) = p_0(\xi)$ in (3.4), we write the value of k_3^- as

$$k_3^- = -\sqrt{\frac{2}{\pi d}}\int_0^{2a}\left(\mathrm{cth}\,\frac{\pi\xi}{d} - \mathrm{cth}\,\frac{2\pi a}{d}\right)^{1/2} p_0(\xi)\,d\xi \tag{3.7}$$

As $a \to \infty$, we find the value of the intensity coefficient k_3 for a periodic system of semi-infinite parallel cracks from (3.7) [9].

In the general case of crack orientation, the solution of (3.1) can be obtained as a power series in λ for large spacings between the cracks. We obtain for the intensity coefficient (the quantities G_s are defined by (2.5))

$$k_3^{\pm} = \sqrt{a}\left\{-\frac{1}{\pi a}\int_{-a}^{a}\sqrt{\frac{a \pm \xi}{a \mp \xi}}\,p(\xi)\,d\xi + \lambda^2 b_1 G_0 + \right.$$

$$\left.\lambda^4\left[\left(2b_2 - \frac{1}{2}b_1^2\right)G_0 + b_2 G_2 \mp \frac{3}{2}b_2 G_1\right]\right\} + O(\lambda^6)$$

$$b_1 = -\frac{\pi^2}{12}\cos 2\alpha, \qquad b_2 = -\frac{\pi^4}{720}\cos 4\alpha$$

In the case of the constant load $p(x) = -\tau$ on the cracks, we find

$$k_3^{\pm} = \tau\sqrt{a}\left[1 + \frac{\pi^2\lambda^2}{24}\cos 2\alpha + \frac{\pi^4\lambda^4}{5760}(28\cos^2 2\alpha - 9)\right] + O(\lambda^6)$$

4. Let us examine a system of N longitudinal shear cracks in an elastic half-space whose surface is load-free. The stress function $F_4(z)$ for such a problem can be obtained from (2.2) by assuming that there are N slits in the upper and lower half-spaces, where the (x, z)-plane is the plane of geometric and force symmetry

$$F_4(z) = \frac{1}{\pi i} \sum_{k=1}^{N} \int_{-a_k}^{a_k} g_k'(t) \left(\frac{1}{te^{i\alpha_k} - z + z_k^\circ} - \frac{1}{te^{-i\alpha_k} - z + \bar{z}_k^\circ} \right) dt \qquad (4.1)$$

Equating the stresses on the surfaces of the cracks to a given load (1.3), we obtain a system of integral equations to determine the unknown functions $g_k'(t)$

$$\int_{-a_n}^{a_n} \frac{g_n'(t)\,dt}{t-x} + \sum_{k=1}^{N} \int_{-a_k}^{a_k} R_{nk}(t,x)\, g_k'(t)\,dt = \pi p_n(x), \; |x| \leqslant a_n \; n = 1,2,\ldots,N \qquad (4.2)$$

$$R_{nk}(t,x) = (1 - \delta_{nk})\, K_{nk}(t,x) + \mathrm{Re}\left(\frac{e^{i\alpha_n}}{X_n - \bar{T}_k} \right)$$

Here δ_{nk} is the Kronecker symbol. The second members in the kernels of (4.2) determine the influence of the free surface of the half-space.

We assume that the centers of all the slits are arranged on one straight line $y = -h$ parallel to the half-space boundary, and the spacing between the centers of adjacent slits is a constant d $(z_k^\circ = kd - ih,\ k = 0, \pm 1, \pm 2, \ldots)$, the lengths and angles of slope of the slits are identical $(a_k = a, \alpha_k = \alpha)$. Considering the same load $(p_k(x) - p(x))$ to be applied to all the slits and their number to tend to infinity, we obtain the stress function $F_5(z)$ for a periodic system of slits in a half-space with a free surface from (4.1)

$$F_5(z) = \frac{1}{id} \int_{-a}^{a} \left[\mathrm{ctg}\, \frac{\pi}{d} (te^{i\alpha} - z - ih) - \mathrm{ctg}\, \frac{\pi}{d} (te^{-i\alpha} - z + ih) \right] g'(t)\,dt$$

We find the integral equation of the problem under consideration by satisfying the boundary condition on the surface of any of the slits

$$\int_{-a}^{a} g'(t)\, R(t,x)\,dt = \pi p(x), \qquad |x| \leqslant a \qquad (4.3)$$

$$R(t,x) = K(t-x) + \frac{\pi}{d}\, \mathrm{Re}\left[e^{i\alpha}\, \mathrm{ctg}\, \frac{\pi}{d} (xe^{i\alpha} - te^{-i\alpha} - 2ih) \right]$$

Let us note that the solution of (4.2) and (4.3) in the case of large spacings between the cracks and the half-space boundaries can be found by the same means as the solution of the system (2.3) has been found.

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OPTIMAL CONTROL OF CERTAIN QUASILINEAR STOCHASTIC SYSTEMS

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The problem of optimal control of quasilinear systems in the presence of external random while noise-type perturbations is considered. Consecutive approximations to the optimal control are obtained and the errors along the trajectory and the optimal functional are estimated.

1. A number of papers in the field of optimal control of stochastic systems which have recently appeared deal with the study of controlled systems containing small terms. This can be explained, in particular, by the fact that although the basic formulations of the problems of stochastic control have been known for considerable time [1, 2], however conclusive results could only be obtained for the linear systems and a quadratic functional. A problem arises of constructing an approximate optimal control by expansion in the terms of a small parameter. For the case when the external perturbations are of low intensity, i.e. when a specified controlled system plays the part of the generating system, the problem of synthesizing an approximate control is dealt with in [3–5] where it is assumed that the solution of the problem is known, and has been obtained in the form of a synthesis.

Another approach to the problem of approximate synthesis of an optimal control is also